Equivalent frame method (EFM) is widely used for the nonlinear static and dynamic analyses in the engineering applications for seismic assessment and design verification of masonry buildings due to its efficiency in the performance prediction and high computational ease. The available large data base on the nonlinear response of masonry elements makes EFM as a suitable choice for the analysis and an ideal option when large building stock and/or parametric sensitivity analyses are under consideration. The available constitutive laws for masonry elements, generally recommended, are investigated in the prediction of seismic displacement demand on the buildings using nonlinear time history analysis (NLTHA) against the analytical prediction using secant vibration period and overdamped spectrum with the bias in the prediction being quantified. The investigation is performed on two to five storey masonry buildings of Pakistan using six different available frame elements constitutive laws. The investigation shows that the bi-linear constitutive law, Takeda type rule, with Emori type of unloading give relatively more consistent results. The effect of different spectral reduction factors, recommended by Euro Code 8, is also investigated.

**Keywords**: Equivalent frame method, constitutive law, seismic assessment, masonry buildings, Pakistan.

### 1. Introduction

The paper presents the investigation of frame-elements force-displacement constitutive laws for nonlinear static and dynamic analyses of masonry buildings using macro-element approach particularly EFM within the context of seismic assessment and design verification of masonry buildings. The investigation is performed using nonlinear dynamic time history analyses (NLTHA) of structural models using real accelerograms in light of the analytical prediction through secant vibration period of the system and overdamped seismic displacement spectrum with the dispersion in the error being quantified.

The displacement-demand on the system is used as the key parameter in the investigation process due to its direct correlation with the structural expected performance level for a given seismic demand (Priestley et al. 2007). The investigation aim at the identification of a conceptual and consistent force-displacement constitutive law for nonlinear static and dynamic analyses of masonry buildings.

### 2. Nonlinear Static and Dynamic Seismic Analyses of Masonry Buildings (SD-SAM)

#### 2.1. Equivalent frame method

In this method the masonry buildings are idealized as an equivalent frame with beams and columns, with material nonlinearity considered through the use of lumped
inelastic hinges, representing the walls and spandrels of the corresponding masonry which is computationally appealing and practically simple to employ. Many macroelement approaches are available in the literature for seismic analysis of masonry buildings using equivalent frame method, the present paper considered the approach SD-SAM proposed and employ by Ahmad et al. (2010a, 2010b and 2010c), Figure 1, due to its simplicity in masonry modeling and efficiency in the analyses.

### 2.2. Frame-elements constitutive law

The frame elements are assigned with the strength-deformability, particularly force-displacement constitutive law, through the definition of inelastic hinges, depending on the element ultimate governing mechanism.

Typical response mechanisms for masonry walls/piers are considered as flexure or rocking response, diagonal shear cracking and shear sliding failure. For spandrels, two response mechanisms are considered i.e. the flexure response in case of any horizontal tension resisting element, e.g. tie rods, and shear response when spandrels are effectively bonded at the ends and having beams above and below the spandrels.

The strength-deformability models for these mechanisms are presented and discussed in Ahmad et al. (2010a, 2010b) from the available literature.

### 3. Displacement-Based Seismic Performance Assessment of Buildings

#### 3.1. Nonlinear static single degree of freedom (SDOF) system for masonry buildings

The seismic response of masonry buildings either with reinforced concrete slab, ring/bond beams, lintel beams, is mainly governed by the global mechanism primarily with shear response of the in-plane walls with limited energy dissipation capabilities and ductility (Javed et al. 2006, Ali and Naeem 2007).

The displacement-based method uses an equivalent nonlinear static SDOF system, called mechanical model in order to simulate analytically the response of an actual structure in terms of its displacement capacity, energy dissipation and secant vibration period. See Figure 2 for the nonlinear behavior of masonry buildings (capacity curve), its idealization as bi-linear or tri-linear SDOF system, limit states ($\Delta y$: $LS_1$, $\Delta m$: $LS_2$, $\Delta u$: $LS_3$) and performance levels ($P_1$: no/slight-damage, $P_2$: moderate damage; $P_3$: heavy damage, $P_4$: collapse).

In this figure, $F_u$ represents the ultimate force; $F_y = 0.9F_u$ represents idealized yielding; $F_{cr} = 0.6F_u$ represents cracking of masonry elements.

Such idealization of masonry buildings is based on the lateral response of individual masonry elements which is relatively well.
represented in case of tri-linear element force-displacement response (Ahmad et al. 2010b).

For the bilinear idealization, the yield displacement do not have a physical meaning but is utilized for the computational efficiency and energy balance criterion (Tomazević, 1999; Magenes and Calvi, 1997). The tri-linear capacity curve is computationally expensive as additional parameters will be required for the post-crack limit states due to hardening and softening branches.

3.2. Determination of seismic displacement demand on the SDOF system

The mechanical model is employed along with the overdamped displacement response spectrum, for a given earthquake, to assess the expected performance level of the building, depending on the pre-specified performance levels, once the displacement-demand on the system is determined, see Figure 3.

Comparing SD with the limit states displacement capacities the expected performance level of the system can be predicted, see Figure 2.

4. Case Study Buildings

4.1. Structural characteristics and modeling

For the case study application, the existing urban brick masonry buildings of Pakistan are considered due to its wide-spread use in the country and the availability of detailed material properties for this typologies. 2D structural models are designed with the site-specific material properties, geometric detailing, loading condition, shear mechanism of masonry elements and soft storey. The nonlinearity is considered only for the masonry walls while the response of span-drels are considered to be elastic. Such consideration meet the requirements of the urban building stock of Pakistan.

4.2. Parametric study

Six different force-displacement constitutive laws: two cases for bi-linear, Takeda type rule, with different unloading stiffness, one for tri-linear and three cases for origin centered, are considered for the investigation.
The analyses are performed using the OpenSeesv2.1.0 (McKenna et al. 2008) and using real accelerograms extracted from the PEER NGA data base on soft soil, previously selected by Pampanin et al. (2002) and employed for masonry by Menon and Magenes (2011) and Ahmad et al. (2010c).

The nonlinear hysteretic model, uniaxial hysteretic material available in OpenSees2.1.0, considered for the masonry walls is modified thin Takeda type rule with Emori unloading stiffness, Figure 1 (right) for shear failure, with $\beta$: 0.5 and 0.6 for bi-linear (TT), 0.6 for tri-linear (TTT) and 1.0 for origin centered (OC). In case of origin centered law, three different options of elastic damping with 5%, 10% and 15% are considered.

5. Results and Discussion

5.1. Equivalent displacement demand on the system from NLTHA and analytical approach

NLTHA is performed on the considered structural models with linearly scaled accelerograms, sixty in total, enough to develop nonlinear behavior of masonry walls up to the ultimate limit state.

The floor displacements for a given accelerogram is normalized by the building deformed shape and seismic mass participation in order to transfer the structural system to the equivalent SDOF system and obtain the displacement demand at the center of seismic force of the system using the approach proposed in Ahmad et al. 2010a, 2010b:

$$\text{SD}_{\text{NLTHA}} = \frac{\sum_{i=1}^{n} m_i \cdot \Delta_i^2}{\sum_{i=1}^{n} m_i \cdot \Delta_i}$$ (1)

where $\text{SD}_{\text{NLTHA}}$ represents the equivalent displacement demand on the equivalent SDOF system; $m_i$ represents the floor mass; $\Delta_i$ represents the floor displacement demand.

Each of the accelerograms is considered, for which elastic 5% damped displacement response spectrum is developed, and analyzed following the procedure depicted in Figure 3 in order to compute the displacement demand on the SDOF system analytically using the material viscous damping proposed by Ahmad et al. 2010b and spectral reduction factor proposed by EC8 (CEN 1994, 2004):

$$\xi = 5 + \frac{32 \cdot (\mu - 1)}{\mu \cdot \pi}$$ (2)

$$\eta_{\text{old}} = \sqrt{\frac{7}{2 + \xi}}$$ (3)

$$\eta_{\text{new}} = \sqrt{\frac{10}{5 + \xi}}$$ (4)

where $\xi$ represents viscous damping of the system; $\eta$ represents spectral reduction factor.

5.2. Estimating dispersion in the prediction

The bias in the prediction of displacement demand is obtained by establishing an empirical relationship between the analytical and NLTHA prediction using the form $\text{EDP} = a \cdot \text{IM}$, where EDP and IM represent the displacement demand on systems from the NLTHA and analytical prediction respectively. The coefficient $a$ is obtained through the regression analysis of the data. The dispersion in the predictions from both the procedures is obtained from the standard deviation of the logarithmic difference i.e. error: $\ln(\text{EDP}) - \ln(a \cdot \text{IM})$, see exemplificative plot Figure 4 for all the considered buildings (two to five storey) with sixty accelerograms for each structural model, 240 cases total.

Such analyses are performed for all the considered nonlinear force-displacement constitutive laws and all the cases study structural models and the results are reported in Table 1. It can be observed, that the bi-linear rules predict the response relatively more consistent with the analytical prediction. The trilinear and origin centered rule predict the response relatively less
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Fig. 4. EDP-IM comparison, development of empirical function and quantifying the dispersion in prediction for the case of TT with $\beta = 0.5$.

$\beta = 0.2506$

$\text{EDP}=1.23\text{IM}$

Table 1. Dispersion in the displacement demand prediction using different constitutive laws.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Force-displacement constitutive law</th>
<th>Damping equation, Eq. (2)</th>
<th>$a$</th>
<th>Std.</th>
<th>$a$</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TT_1</td>
<td>EC8$_{\text{old}}$</td>
<td>1.2301</td>
<td>0.2506</td>
<td>1.1776</td>
<td>0.313</td>
</tr>
<tr>
<td>2</td>
<td>TT_2</td>
<td>EC8$_{\text{old}}$</td>
<td>1.2849</td>
<td>0.2451</td>
<td>1.2282</td>
<td>0.3021</td>
</tr>
<tr>
<td>3</td>
<td>TTT</td>
<td>EC8$_{\text{new}}$</td>
<td>1.6859</td>
<td>0.3242</td>
<td>1.6117</td>
<td>0.3672</td>
</tr>
<tr>
<td>4</td>
<td>OC_1</td>
<td>EC8$_{\text{old}}$</td>
<td>3.4512</td>
<td>0.6482</td>
<td>3.3199</td>
<td>0.6664</td>
</tr>
<tr>
<td>5</td>
<td>OC_2</td>
<td>EC8$_{\text{old}}$</td>
<td>3.0924</td>
<td>0.6473</td>
<td>2.9606</td>
<td>0.6546</td>
</tr>
<tr>
<td>6</td>
<td>OC_3</td>
<td>EC8$_{\text{new}}$</td>
<td>2.6717</td>
<td>0.6012</td>
<td>2.5455</td>
<td>0.607</td>
</tr>
</tbody>
</table>

The dispersion in the response prediction is obtained to be relatively less for the old EC8 Eq. (3) for spectral reduction, however the offset in the prediction reduces using the new EC8 recommended spectral reduction Eq. (4). One point worthy to be mentioned herein that the present analytical static analysis considered the bi-linear idealization of the global response of the masonry buildings, which is nevertheless recommended by the structural and masonry material experts following many experimental and numerical investigations (Tomazevic, 1999; Magenes and Calvi, 1997). The present investigation has to be performed using more realistic global response of buildings in order to validate the findings reported herein and increase the confidence level in future application.

6. Conclusions

The paper presents the investigation of nonlinear constitutive laws for masonry buildings within the context of seismic assessment and design verification. Six different nonlinear force-displacement rules are used to predict the seismic displacement demand on the structural system using real accelerograms and NLTHA. The seismic demand predicted with NLTHA is compared with the analytical prediction recommended by structural experts for static analysis, using the secant vibration period, material viscous damping and overdamped displacement spectrum, in order to quantify the expected dispersion in the response evaluation using NLTHA. The investigation found herein that bi-linear force-displacement law with Takeda type hysteresis rule and Emori type of unloading predict the response relatively more consistent with the static prediction. The effect of spectral reduction factor proposed by EC8 (old and new) is also investigated which gives less dispersion in the demand prediction for the case of old recommendation but predict the response with less offset using the new version. The crucial assumption made in the present study is the idealization of global response of masonry buildings as elasto-plastic, which is nevertheless recommended by structural and material experts, need to be investigated with more realistic nonlinear response in order to validate the findings reported herein and increase the confidence level in future applications.

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